

Maple 2018.2 Integration Test Results
on the problems in "6 Hyperbolic functions/6.6 Hyperbolic cosecant"

Test results for the 11 problems in "6.6.1 (c+d x)^m (a+b csch)^n.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^3 \operatorname{csch}(bx+a) dx$$

Optimal(type 4, 142 leaves, 9 steps):

$$\begin{aligned} & -\frac{2(dx+c)^3 \operatorname{arctanh}(e^{bx+a})}{b} - \frac{3d(dx+c)^2 \operatorname{polylog}(2, -e^{bx+a})}{b^2} + \frac{3d(dx+c)^2 \operatorname{polylog}(2, e^{bx+a})}{b^2} + \frac{6d^2(dx+c) \operatorname{polylog}(3, -e^{bx+a})}{b^3} \\ & - \frac{6d^2(dx+c) \operatorname{polylog}(3, e^{bx+a})}{b^3} - \frac{6d^3 \operatorname{polylog}(4, -e^{bx+a})}{b^4} + \frac{6d^3 \operatorname{polylog}(4, e^{bx+a})}{b^4} \end{aligned}$$

Result(type 4, 540 leaves):

$$\begin{aligned} & -\frac{d^3 a^3 \ln(1+e^{bx+a})}{b^4} + \frac{d^3 a^3 \ln(1-e^{bx+a})}{b^4} - \frac{d^3 \ln(1+e^{bx+a}) x^3}{b} - \frac{3d^3 \operatorname{polylog}(2, -e^{bx+a}) x^2}{b^2} + \frac{6d^3 \operatorname{polylog}(3, -e^{bx+a}) x}{b^3} + \frac{d^3 \ln(1-e^{bx+a}) x^3}{b} \\ & + \frac{3d^3 \operatorname{polylog}(2, e^{bx+a}) x^2}{b^2} - \frac{6d^3 \operatorname{polylog}(3, e^{bx+a}) x}{b^3} - \frac{3c^2 d \operatorname{polylog}(2, -e^{bx+a})}{b^2} + \frac{3c^2 d \operatorname{polylog}(2, e^{bx+a})}{b^2} + \frac{6cd^2 \operatorname{polylog}(3, -e^{bx+a})}{b^3} \\ & - \frac{6cd^2 \operatorname{polylog}(3, e^{bx+a})}{b^3} + \frac{2d^3 a^3 \operatorname{arctanh}(e^{bx+a})}{b^4} - \frac{2c^3 \operatorname{arctanh}(e^{bx+a})}{b} + \frac{3cd^2 a^2 \ln(1+e^{bx+a})}{b^3} - \frac{3cd^2 a^2 \ln(1-e^{bx+a})}{b^3} \\ & - \frac{3c^2 d \ln(1+e^{bx+a}) x}{b} - \frac{3c^2 d \ln(1+e^{bx+a}) a}{b^2} + \frac{3c^2 d \ln(1-e^{bx+a}) x}{b} + \frac{3c^2 d \ln(1-e^{bx+a}) a}{b^2} - \frac{3cd^2 \ln(1+e^{bx+a}) x^2}{b} \\ & - \frac{6cd^2 \operatorname{polylog}(2, -e^{bx+a}) x}{b^2} + \frac{3cd^2 \ln(1-e^{bx+a}) x^2}{b} + \frac{6cd^2 \operatorname{polylog}(2, e^{bx+a}) x}{b^2} - \frac{6d^2 a^2 c \operatorname{arctanh}(e^{bx+a})}{b^3} + \frac{6da c^2 \operatorname{arctanh}(e^{bx+a})}{b^2} \\ & - \frac{6d^3 \operatorname{polylog}(4, -e^{bx+a})}{b^4} + \frac{6d^3 \operatorname{polylog}(4, e^{bx+a})}{b^4} \end{aligned}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^2 \operatorname{csch}(bx+a)^3 dx$$

Optimal(type 4, 147 leaves, 9 steps):

$$\begin{aligned} & \frac{(dx+c)^2 \operatorname{arctanh}(e^{bx+a})}{b} - \frac{d^2 \operatorname{arctanh}(\cosh(bx+a))}{b^3} - \frac{d(dx+c) \operatorname{csch}(bx+a)}{b^2} - \frac{(dx+c)^2 \operatorname{coth}(bx+a) \operatorname{csch}(bx+a)}{2b} \\ & + \frac{d(dx+c) \operatorname{polylog}(2, -e^{bx+a})}{b^2} - \frac{d(dx+c) \operatorname{polylog}(2, e^{bx+a})}{b^2} - \frac{d^2 \operatorname{polylog}(3, -e^{bx+a})}{b^3} + \frac{d^2 \operatorname{polylog}(3, e^{bx+a})}{b^3} \end{aligned}$$

Result(type 4, 443 leaves):

$$\begin{aligned}
& - \frac{e^{bx+a} (bd^2x^2 e^{2bx+2a} + 2bcdxe^{2bx+2a} + bc^2 e^{2bx+2a} + bd^2x^2 + 2d^2xe^{2bx+2a} + 2bcdx + 2cde^{2bx+2a} + bc^2 - 2d^2x - 2cd)}{b^2 (e^{2bx+2a} - 1)^2} \\
& - \frac{2acd \operatorname{arctanh}(e^{bx+a})}{b^2} + \frac{\ln(1 + e^{bx+a}) cdx}{b} + \frac{\ln(1 + e^{bx+a}) acd}{b^2} - \frac{\ln(1 - e^{bx+a}) cdx}{b} - \frac{\ln(1 - e^{bx+a}) acd}{b^2} - \frac{d^2 \operatorname{polylog}(3, -e^{bx+a})}{b^3} \\
& + \frac{d^2 \operatorname{polylog}(3, e^{bx+a})}{b^3} - \frac{2d^2 \operatorname{arctanh}(e^{bx+a})}{b^3} + \frac{cd \operatorname{polylog}(2, -e^{bx+a})}{b^2} - \frac{cd \operatorname{polylog}(2, e^{bx+a})}{b^2} - \frac{\ln(1 + e^{bx+a}) a^2 d^2}{2b^3} + \frac{\ln(1 - e^{bx+a}) a^2 d^2}{2b^3} \\
& + \frac{a^2 d^2 \operatorname{arctanh}(e^{bx+a})}{b^3} + \frac{c^2 \operatorname{arctanh}(e^{bx+a})}{b} + \frac{\ln(1 + e^{bx+a}) d^2 x^2}{2b} + \frac{\operatorname{polylog}(2, -e^{bx+a}) d^2 x}{b^2} - \frac{\ln(1 - e^{bx+a}) d^2 x^2}{2b} - \frac{\operatorname{polylog}(2, e^{bx+a}) d^2 x}{b^2}
\end{aligned}$$

Problem 5: Unable to integrate problem.

$$\int \left(\frac{x}{\operatorname{csch}(x)^{7/2}} - \frac{5x\sqrt{\operatorname{csch}(x)}}{21} \right) dx$$

Optimal(type 3, 31 leaves, 5 steps):

$$-\frac{4}{49 \operatorname{csch}(x)^{7/2}} + \frac{2x \cosh(x)}{7 \operatorname{csch}(x)^{5/2}} + \frac{20}{63 \operatorname{csch}(x)^{3/2}} - \frac{10x \cosh(x)}{21 \sqrt{\operatorname{csch}(x)}}$$

Result(type 8, 16 leaves):

$$\int \left(\frac{x}{\operatorname{csch}(x)^{7/2}} - \frac{5x\sqrt{\operatorname{csch}(x)}}{21} \right) dx$$

Problem 6: Unable to integrate problem.

$$\int \left(\frac{x^2}{\operatorname{csch}(x)^{3/2}} + \frac{x^2\sqrt{\operatorname{csch}(x)}}{3} \right) dx$$

Optimal(type 4, 78 leaves, 7 steps):

$$-\frac{8x}{9 \operatorname{csch}(x)^{3/2}} + \frac{16 \cosh(x)}{27 \sqrt{\operatorname{csch}(x)}} + \frac{2x^2 \cosh(x)}{3 \sqrt{\operatorname{csch}(x)}} - \frac{16I \sqrt{\sin\left(\frac{\pi}{4} + \frac{Ix}{2}\right)^2} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{Ix}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(x)} \sqrt{I \sinh(x)}}{27 \sin\left(\frac{\pi}{4} + \frac{Ix}{2}\right)}$$

Result(type 8, 20 leaves):

$$\int \left(\frac{x^2}{\operatorname{csch}(x)^{3/2}} + \frac{x^2\sqrt{\operatorname{csch}(x)}}{3} \right) dx$$

Problem 7: Unable to integrate problem.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{a + b \operatorname{csch}(dx + c)} dx$$

Optimal(type 4, 422 leaves, 17 steps):

$$\begin{aligned}
& \frac{b (fx + e)^4}{4 a^2 f} - \frac{6 f^3 \cosh(dx + c)}{a d^4} - \frac{3 f (fx + e)^2 \cosh(dx + c)}{a d^2} - \frac{b (fx + e)^3 \ln\left(1 + \frac{a e^{dx+c}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 d} - \frac{b (fx + e)^3 \ln\left(1 + \frac{a e^{dx+c}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 d} \\
& - \frac{3 b f (fx + e)^2 \operatorname{polylog}\left(2, -\frac{a e^{dx+c}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 d^2} - \frac{3 b f (fx + e)^2 \operatorname{polylog}\left(2, -\frac{a e^{dx+c}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 d^2} + \frac{6 b f^2 (fx + e) \operatorname{polylog}\left(3, -\frac{a e^{dx+c}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 d^3} \\
& + \frac{6 b f^2 (fx + e) \operatorname{polylog}\left(3, -\frac{a e^{dx+c}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 d^3} - \frac{6 b f^3 \operatorname{polylog}\left(4, -\frac{a e^{dx+c}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 d^4} - \frac{6 b f^3 \operatorname{polylog}\left(4, -\frac{a e^{dx+c}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 d^4} \\
& + \frac{6 f^2 (fx + e) \sinh(dx + c)}{a d^3} + \frac{(fx + e)^3 \sinh(dx + c)}{a d}
\end{aligned}$$

Result(type 8, 368 leaves):

$$\begin{aligned}
& - \frac{b \left(\frac{1}{4} f^3 x^4 + e f^2 x^3 + \frac{3}{2} e^2 f x^2 + e^3 x\right)}{a^2} + \frac{(f^3 x^3 d^3 + 3 d^3 e f^2 x^2 + 3 d^3 e^2 f x - 3 d^2 f^3 x^2 + d^3 e^3 - 6 d^2 e f^2 x - 3 e^2 f d^2 + 6 d f^3 x + 6 e f^2 d - 6 f^3) e^{dx+c}}{2 a d^4} \\
& - \frac{f^3 x^3 d^3 + 3 d^3 e f^2 x^2 + 3 d^3 e^2 f x + 3 d^2 f^3 x^2 + d^3 e^3 + 6 d^2 e f^2 x + 3 e^2 f d^2 + 6 d f^3 x + 6 e f^2 d + 6 f^3}{2 a d^4 e^{dx+c}} + \int \\
& - \frac{2 b (-b f^3 x^3 e^{dx+c} + a f^3 x^3 - 3 b e f^2 x^2 e^{dx+c} + 3 a e f^2 x^2 - 3 b e^2 f x e^{dx+c} + 3 a e^2 f x - b e^3 e^{dx+c} + a e^3)}{(a (e^{dx+c})^2 + 2 b e^{dx+c} - a) a^2} dx
\end{aligned}$$

Problem 8: Unable to integrate problem.

$$\int \frac{(fx + e)^2 \cosh(dx + c)}{a + b \operatorname{csch}(dx + c)} dx$$

Optimal(type 4, 310 leaves, 14 steps):

$$\begin{aligned}
& \frac{b (fx + e)^3}{3 a^2 f} - \frac{2 f (fx + e) \cosh(dx + c)}{a d^2} - \frac{b (fx + e)^2 \ln\left(1 + \frac{a e^{dx+c}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 d} - \frac{b (fx + e)^2 \ln\left(1 + \frac{a e^{dx+c}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 d} \\
& - \frac{2 b f (fx + e) \operatorname{polylog}\left(2, -\frac{a e^{dx+c}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 d^2} - \frac{2 b f (fx + e) \operatorname{polylog}\left(2, -\frac{a e^{dx+c}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 d^2} + \frac{2 b f^2 \operatorname{polylog}\left(3, -\frac{a e^{dx+c}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 d^3} \\
& + \frac{2 b f^2 \operatorname{polylog}\left(3, -\frac{a e^{dx+c}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 d^3} + \frac{2 f^2 \sinh(dx + c)}{a d^3} + \frac{(fx + e)^2 \sinh(dx + c)}{a d}
\end{aligned}$$

Result(type 8, 235 leaves):

$$-\frac{b \left(\frac{1}{3} f^2 x^3 + f e x^2 + e^2 x \right)}{a^2} + \frac{(f^2 x^2 d^2 + 2 d^2 e f x + d^2 e^2 - 2 d f^2 x - 2 e f d + 2 f^2) e^{d x + c}}{2 a d^3} - \frac{f^2 x^2 d^2 + 2 d^2 e f x + d^2 e^2 + 2 d f^2 x + 2 e f d + 2 f^2}{2 a d^3 e^{d x + c}} + \int \frac{-2 b (-b f^2 x^2 e^{d x + c} + a f^2 x^2 - 2 b e f x e^{d x + c} + 2 a e f x - b e^2 e^{d x + c} + a e^2)}{(a (e^{d x + c})^2 + 2 b e^{d x + c} - a) a^2} dx$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x + e) \cosh(d x + c)}{a + b \operatorname{csch}(d x + c)} dx$$

Optimal(type 4, 198 leaves, 11 steps):

$$\frac{b (f x + e)^2}{2 a^2 f} - \frac{f \cosh(d x + c)}{a d^2} - \frac{b (f x + e) \ln \left(1 + \frac{a e^{d x + c}}{b - \sqrt{a^2 + b^2}} \right)}{a^2 d} - \frac{b (f x + e) \ln \left(1 + \frac{a e^{d x + c}}{b + \sqrt{a^2 + b^2}} \right)}{a^2 d} - \frac{b f \operatorname{polylog} \left(2, -\frac{a e^{d x + c}}{b - \sqrt{a^2 + b^2}} \right)}{a^2 d^2} - \frac{b f \operatorname{polylog} \left(2, -\frac{a e^{d x + c}}{b + \sqrt{a^2 + b^2}} \right)}{a^2 d^2} + \frac{(f x + e) \sinh(d x + c)}{a d}$$

Result(type 4, 482 leaves):

$$\frac{b f x^2}{2 a^2} - \frac{b e x}{a^2} + \frac{(f x d + e d - f) e^{d x + c}}{2 d^2 a} - \frac{(f x d + e d + f) e^{-d x - c}}{2 d^2 a} - \frac{b e \ln(a e^{2 d x + 2 c} + 2 b e^{d x + c} - a)}{a^2 d} + \frac{2 b e \ln(e^{d x + c})}{a^2 d} - \frac{b f \ln \left(\frac{-a e^{d x + c} + \sqrt{a^2 + b^2} - b}{-b + \sqrt{a^2 + b^2}} \right) x}{a^2 d} - \frac{b f \ln \left(\frac{-a e^{d x + c} + \sqrt{a^2 + b^2} - b}{-b + \sqrt{a^2 + b^2}} \right) c}{a^2 d^2} - \frac{b f \ln \left(\frac{a e^{d x + c} + \sqrt{a^2 + b^2} + b}{b + \sqrt{a^2 + b^2}} \right) x}{a^2 d} - \frac{b f \ln \left(\frac{a e^{d x + c} + \sqrt{a^2 + b^2} + b}{b + \sqrt{a^2 + b^2}} \right) c}{a^2 d^2} - \frac{b f \operatorname{dilog} \left(\frac{a e^{d x + c} + \sqrt{a^2 + b^2} + b}{b + \sqrt{a^2 + b^2}} \right)}{a^2 d^2} - \frac{b f \operatorname{dilog} \left(\frac{-a e^{d x + c} + \sqrt{a^2 + b^2} - b}{-b + \sqrt{a^2 + b^2}} \right)}{a^2 d^2} + \frac{2 b f c x}{a^2 d} + \frac{b f c^2}{a^2 d^2} + \frac{b f c \ln(a e^{2 d x + 2 c} + 2 b e^{d x + c} - a)}{a^2 d^2} - \frac{2 b f c \ln(e^{d x + c})}{a^2 d^2}$$

Problem 10: Unable to integrate problem.

$$\int \frac{(f x + e)^2 \cosh(d x + c)^2}{a + b \operatorname{csch}(d x + c)} dx$$

Optimal(type 4, 468 leaves, 21 steps):

$$\begin{aligned}
& \frac{f^2 x}{4 a d^2} + \frac{(f x + e)^3}{6 a f} + \frac{b^2 (f x + e)^3}{3 a^3 f} - \frac{2 b f^2 \cosh(d x + c)}{a^2 d^3} - \frac{b (f x + e)^2 \cosh(d x + c)}{a^2 d} - \frac{f (f x + e) \cosh(d x + c)^2}{2 a d^2} + \frac{2 b f (f x + e) \sinh(d x + c)}{a^2 d^2} \\
& + \frac{f^2 \cosh(d x + c) \sinh(d x + c)}{4 a d^3} + \frac{(f x + e)^2 \cosh(d x + c) \sinh(d x + c)}{2 a d} - \frac{b (f x + e)^2 \ln\left(1 + \frac{a e^{d x + c}}{b - \sqrt{a^2 + b^2}}\right) \sqrt{a^2 + b^2}}{a^3 d} \\
& + \frac{b (f x + e)^2 \ln\left(1 + \frac{a e^{d x + c}}{b + \sqrt{a^2 + b^2}}\right) \sqrt{a^2 + b^2}}{a^3 d} - \frac{2 b f (f x + e) \operatorname{polylog}\left(2, -\frac{a e^{d x + c}}{b - \sqrt{a^2 + b^2}}\right) \sqrt{a^2 + b^2}}{a^3 d^2} \\
& + \frac{2 b f (f x + e) \operatorname{polylog}\left(2, -\frac{a e^{d x + c}}{b + \sqrt{a^2 + b^2}}\right) \sqrt{a^2 + b^2}}{a^3 d^2} + \frac{2 b f^2 \operatorname{polylog}\left(3, -\frac{a e^{d x + c}}{b - \sqrt{a^2 + b^2}}\right) \sqrt{a^2 + b^2}}{a^3 d^3} \\
& - \frac{2 b f^2 \operatorname{polylog}\left(3, -\frac{a e^{d x + c}}{b + \sqrt{a^2 + b^2}}\right) \sqrt{a^2 + b^2}}{a^3 d^3}
\end{aligned}$$

Result(type 8, 391 leaves):

$$\begin{aligned}
& \frac{\frac{1}{3} a^2 f^2 x^3 + \frac{2}{3} b^2 f^2 x^3 + a^2 e f x^2 + 2 b^2 e f x^2 + a^2 e^2 x + 2 b^2 e^2 x}{2 a^3} + \frac{(2 f^2 x^2 d^2 + 4 d^2 e f x + 2 d^2 e^2 - 2 d f^2 x - 2 e f d + f^2) (e^{d x + c})^2}{16 d^3 a} \\
& - \frac{b (f^2 x^2 d^2 + 2 d^2 e f x + d^2 e^2 - 2 d f^2 x - 2 e f d + 2 f^2) e^{d x + c}}{2 a^2 d^3} - \frac{b (f^2 x^2 d^2 + 2 d^2 e f x + d^2 e^2 + 2 d f^2 x + 2 e f d + 2 f^2)}{2 a^2 d^3 e^{d x + c}} \\
& - \frac{2 f^2 x^2 d^2 + 4 d^2 e f x + 2 d^2 e^2 + 2 d f^2 x + 2 e f d + f^2}{16 d^3 a (e^{d x + c})^2} + \int -\frac{2 b (a^2 f^2 x^2 + b^2 f^2 x^2 + 2 a^2 e f x + 2 b^2 e f x + a^2 e^2 + b^2 e^2) e^{d x + c}}{(a (e^{d x + c})^2 + 2 b e^{d x + c} - a) a^3} dx
\end{aligned}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(d x + c)^3}{a + b \operatorname{csch}(d x + c)} dx$$

Optimal(type 3, 81 leaves, 5 steps):

$$-\frac{b (a^2 + b^2) \ln(b + a \sinh(d x + c))}{a^4 d} + \frac{(a^2 + b^2) \sinh(d x + c)}{a^3 d} - \frac{b \sinh(d x + c)^2}{2 a^2 d} + \frac{\sinh(d x + c)^3}{3 a d}$$

Result(type 3, 427 leaves):

$$-\frac{1}{3 d a \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right) - 1\right)^3} - \frac{b}{2 d a^2 \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{1}{2 d a \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{1}{d a \left(\tanh\left(\frac{d x}{2} + \frac{c}{2}\right) - 1\right)}$$

$$\begin{aligned}
& -\frac{b}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{b^2}{da^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{b \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da^2} + \frac{b^3 \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da^4} \\
& - \frac{b \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 b - 2a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - b}{da^2} - \frac{b^3 \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 b - 2a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - b}{da^4} - \frac{1}{3da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} \\
& + \frac{1}{2da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{b}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{1}{da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{b}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} \\
& - \frac{b^2}{da^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{b \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da^2} + \frac{b^3 \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da^4}
\end{aligned}$$

Test results for the 25 problems in "6.6.2 (e x)^m (a+b csch(c+d x^n))^p.txt"

Problem 1: Unable to integrate problem.

$$\int x^5 (a + b \operatorname{csch}(dx^2 + c)) dx$$

Optimal(type 4, 97 leaves, 10 steps):

$$\frac{ax^6}{6} - \frac{bx^4 \operatorname{arctanh}(e^{dx^2+c})}{d} - \frac{bx^2 \operatorname{polylog}(2, -e^{dx^2+c})}{d^2} + \frac{bx^2 \operatorname{polylog}(2, e^{dx^2+c})}{d^2} + \frac{b \operatorname{polylog}(3, -e^{dx^2+c})}{d^3} - \frac{b \operatorname{polylog}(3, e^{dx^2+c})}{d^3}$$

Result(type 8, 37 leaves):

$$\frac{ax^6}{6} + \int \frac{2e^{dx^2+c}bx^5}{(e^{dx^2+c})^2 - 1} dx$$

Problem 2: Unable to integrate problem.

$$\int x^3 (a + b \operatorname{csch}(dx^2 + c)) dx$$

Optimal(type 4, 59 leaves, 8 steps):

$$\frac{ax^4}{4} - \frac{bx^2 \operatorname{arctanh}(e^{dx^2+c})}{d} - \frac{b \operatorname{polylog}(2, -e^{dx^2+c})}{2d^2} + \frac{b \operatorname{polylog}(2, e^{dx^2+c})}{2d^2}$$

Result(type 8, 37 leaves):

$$\frac{ax^4}{4} + \int \frac{2e^{dx^2+c}bx^3}{(e^{dx^2+c})^2 - 1} dx$$

Problem 4: Unable to integrate problem.

$$\int x^3 (a + b \operatorname{csch}(dx^2 + c))^2 dx$$

Optimal(type 4, 99 leaves, 10 steps):

$$\frac{a^2 x^4}{4} - \frac{2 a b x^2 \operatorname{arctanh}(e^{dx^2+c})}{d} - \frac{b^2 x^2 \operatorname{coth}(dx^2+c)}{2d} + \frac{b^2 \ln(\sinh(dx^2+c))}{2d^2} - \frac{a b \operatorname{polylog}(2, -e^{dx^2+c})}{d^2} + \frac{a b \operatorname{polylog}(2, e^{dx^2+c})}{d^2}$$

Result(type 8, 74 leaves):

$$\frac{a^2 x^4}{4} - \frac{b^2 x^2}{d((e^{dx^2+c})^2 - 1)} + \int \frac{2 b x (2 a d x^2 e^{dx^2+c} + b)}{d((e^{dx^2+c})^2 - 1)} dx$$

Problem 6: Unable to integrate problem.

$$\int \frac{x^5}{a + b \operatorname{csch}(dx^2 + c)} dx$$

Optimal(type 4, 289 leaves, 13 steps):

$$\begin{aligned} \frac{x^6}{6a} - \frac{b x^4 \ln\left(1 + \frac{a e^{dx^2+c}}{b - \sqrt{a^2+b^2}}\right)}{2 a d \sqrt{a^2+b^2}} + \frac{b x^4 \ln\left(1 + \frac{a e^{dx^2+c}}{b + \sqrt{a^2+b^2}}\right)}{2 a d \sqrt{a^2+b^2}} - \frac{b x^2 \operatorname{polylog}\left(2, -\frac{a e^{dx^2+c}}{b - \sqrt{a^2+b^2}}\right)}{a d^2 \sqrt{a^2+b^2}} + \frac{b x^2 \operatorname{polylog}\left(2, -\frac{a e^{dx^2+c}}{b + \sqrt{a^2+b^2}}\right)}{a d^2 \sqrt{a^2+b^2}} \\ + \frac{b \operatorname{polylog}\left(3, -\frac{a e^{dx^2+c}}{b - \sqrt{a^2+b^2}}\right)}{a d^3 \sqrt{a^2+b^2}} - \frac{b \operatorname{polylog}\left(3, -\frac{a e^{dx^2+c}}{b + \sqrt{a^2+b^2}}\right)}{a d^3 \sqrt{a^2+b^2}} \end{aligned}$$

Result(type 8, 57 leaves):

$$\frac{x^6}{6a} + \int -\frac{2 e^{dx^2+c} b x^5}{a (a (e^{dx^2+c})^2 + 2 b e^{dx^2+c} - a)} dx$$

Problem 8: Unable to integrate problem.

$$\int \frac{x^3}{a + b \operatorname{csch}(dx^2 + c)} dx$$

Optimal(type 4, 195 leaves, 11 steps):

$$\frac{x^4}{4a} - \frac{b x^2 \ln\left(1 + \frac{a e^{dx^2+c}}{b - \sqrt{a^2+b^2}}\right)}{2 a d \sqrt{a^2+b^2}} + \frac{b x^2 \ln\left(1 + \frac{a e^{dx^2+c}}{b + \sqrt{a^2+b^2}}\right)}{2 a d \sqrt{a^2+b^2}} - \frac{b \operatorname{polylog}\left(2, -\frac{a e^{dx^2+c}}{b - \sqrt{a^2+b^2}}\right)}{2 a d^2 \sqrt{a^2+b^2}} + \frac{b \operatorname{polylog}\left(2, -\frac{a e^{dx^2+c}}{b + \sqrt{a^2+b^2}}\right)}{2 a d^2 \sqrt{a^2+b^2}}$$

Result(type 8, 57 leaves):

$$\frac{x^4}{4a} + \int -\frac{2e^{dx^2+c}bx^3}{a(a(e^{dx^2+c})^2+2be^{dx^2+c}-a)} dx$$

Problem 10: Unable to integrate problem.

$$\int \frac{x^5}{(a+b\operatorname{csch}(dx^2+c))^2} dx$$

Optimal (type 4, 840 leaves, 31 steps):

$$\begin{aligned} & -\frac{b^2x^4}{2a^2(a^2+b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\ln\left(1+\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} + \frac{b^3x^4\ln\left(1+\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d} + \frac{b^2x^2\ln\left(1+\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^2} \\ & -\frac{b^3x^4\ln\left(1+\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{3/2}d} + \frac{b^2\operatorname{polylog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} + \frac{b^3x^2\operatorname{polylog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} + \frac{b^2\operatorname{polylog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^3} \\ & -\frac{b^3x^2\operatorname{polylog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{b^3\operatorname{polylog}\left(3,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3} + \frac{b^3\operatorname{polylog}\left(3,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3} \\ & -\frac{b^2x^4\cosh(dx^2+c)}{2a(a^2+b^2)d(b+a\sinh(dx^2+c))} - \frac{bx^4\ln\left(1+\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2d\sqrt{a^2+b^2}} + \frac{bx^4\ln\left(1+\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2d\sqrt{a^2+b^2}} - \frac{2bx^2\operatorname{polylog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}} \\ & + \frac{2bx^2\operatorname{polylog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}} + \frac{2b\operatorname{polylog}\left(3,-\frac{ae^{dx^2+c}}{b-\sqrt{a^2+b^2}}\right)}{a^2d^3\sqrt{a^2+b^2}} - \frac{2b\operatorname{polylog}\left(3,-\frac{ae^{dx^2+c}}{b+\sqrt{a^2+b^2}}\right)}{a^2d^3\sqrt{a^2+b^2}} \end{aligned}$$

Result (type 8, 177 leaves):

$$\frac{x^6}{6a^2} - \frac{b^2x^4(-be^{dx^2+c}+a)}{a^2(a^2+b^2)d(a(e^{dx^2+c})^2+2be^{dx^2+c}-a)} + \int -\frac{2bx^3(2a^2dx^2e^{dx^2+c}+b^2dx^2e^{dx^2+c}+2b^2e^{dx^2+c}-2ab)}{a^2(a^2+b^2)d(a(e^{dx^2+c})^2+2be^{dx^2+c}-a)} dx$$

Problem 14: Unable to integrate problem.

$$\int \frac{x^3}{a+b\operatorname{csch}(c+d\sqrt{x})} dx$$

Optimal (type 4, 767 leaves, 23 steps):

$$\begin{aligned}
& \frac{x^4}{4a} - \frac{2bx^7/2 \ln\left(1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}} + \frac{2bx^7/2 \ln\left(1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}} - \frac{14bx^3 \operatorname{polylog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} \\
& + \frac{14bx^3 \operatorname{polylog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{84bx^5/2 \operatorname{polylog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{84bx^5/2 \operatorname{polylog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} \\
& - \frac{420bx^2 \operatorname{polylog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}} + \frac{420bx^2 \operatorname{polylog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}} + \frac{1680bx^3/2 \operatorname{polylog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{ad^5\sqrt{a^2+b^2}} \\
& - \frac{1680bx^3/2 \operatorname{polylog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)}{ad^5\sqrt{a^2+b^2}} - \frac{5040bx \operatorname{polylog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{ad^6\sqrt{a^2+b^2}} + \frac{5040bx \operatorname{polylog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)}{ad^6\sqrt{a^2+b^2}} \\
& - \frac{10080b \operatorname{polylog}\left(8, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{ad^8\sqrt{a^2+b^2}} + \frac{10080b \operatorname{polylog}\left(8, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)}{ad^8\sqrt{a^2+b^2}} + \frac{10080b \operatorname{polylog}\left(7, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)\sqrt{x}}{ad^7\sqrt{a^2+b^2}} \\
& - \frac{10080b \operatorname{polylog}\left(7, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)\sqrt{x}}{ad^7\sqrt{a^2+b^2}}
\end{aligned}$$

Result(type 8, 20 leaves):

$$\int \frac{x^3}{a + b \operatorname{csch}(c + d\sqrt{x})} dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{x^2}{(a + b \operatorname{csch}(c + d\sqrt{x}))^2} dx$$

Optimal(type 4, 1723 leaves, 49 steps):

$$-\frac{240b^2 \operatorname{polylog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^6} - \frac{240b^2 \operatorname{polylog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)d^6} + \frac{240b^3 \operatorname{polylog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^3/2d^6}$$

$$\begin{aligned}
& - \frac{240 b^3 \operatorname{polylog}\left(6, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^6} - \frac{480 b \operatorname{polylog}\left(6, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d^6 \sqrt{a^2+b^2}} + \frac{480 b \operatorname{polylog}\left(6, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d^6 \sqrt{a^2+b^2}} - \frac{2 b^2 x^5 / 2}{a^2 (a^2+b^2) d} + \frac{x^3}{3 a^2} \\
& + \frac{40 b^2 x^3 / 2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^3} - \frac{10 b^3 x^2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^2} - \frac{120 b^2 x \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^4} \\
& - \frac{40 b^3 x^3 / 2 \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^3} - \frac{120 b^2 x \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^4} + \frac{40 b^3 x^3 / 2 \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^3} \\
& + \frac{120 b^3 x \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^4} - \frac{120 b^3 x \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^4} - \frac{4 b x^5 / 2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d \sqrt{a^2+b^2}} \\
& + \frac{4 b x^5 / 2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d \sqrt{a^2+b^2}} - \frac{20 b x^2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}} + \frac{20 b x^2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}} \\
& + \frac{80 b x^3 / 2 \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d^3 \sqrt{a^2+b^2}} - \frac{80 b x^3 / 2 \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d^3 \sqrt{a^2+b^2}} - \frac{240 b x \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d^4 \sqrt{a^2+b^2}} \\
& + \frac{240 b x \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d^4 \sqrt{a^2+b^2}} + \frac{240 b^2 \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 (a^2+b^2) d^5} + \frac{240 b^2 \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 (a^2+b^2) d^5} \\
& - \frac{240 b^3 \operatorname{polylog}\left(5, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 (a^2+b^2)^{3/2} d^5} + \frac{240 b^3 \operatorname{polylog}\left(5, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 (a^2+b^2)^{3/2} d^5} + \frac{480 b \operatorname{polylog}\left(5, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 d^5 \sqrt{a^2+b^2}} \\
& - \frac{480 b \operatorname{polylog}\left(5, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 d^5 \sqrt{a^2+b^2}} + \frac{10 b^2 x^2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^2} + \frac{2 b^3 x^5 / 2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d}
\end{aligned}$$

$$\begin{aligned}
& + \frac{10 b^2 x^2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 (a^2 + b^2) d^2} - \frac{2 b^3 x^5 / 2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b + \sqrt{a^2 + b^2}}\right)}{a^2 (a^2 + b^2)^3 / 2 d} + \frac{40 b^2 x^3 / 2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 (a^2 + b^2) d^3} \\
& + \frac{10 b^3 x^2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b - \sqrt{a^2 + b^2}}\right)}{a^2 (a^2 + b^2)^3 / 2 d^2} - \frac{2 b^2 x^5 / 2 \cosh(c + d\sqrt{x})}{a (a^2 + b^2) d (b + a \sinh(c + d\sqrt{x}))}
\end{aligned}$$

Result(type 8, 20 leaves):

$$\int \frac{x^2}{(a + b \operatorname{csch}(c + d\sqrt{x}))^2} dx$$

Problem 18: Unable to integrate problem.

$$\int x^3 / 2 (a + b \operatorname{csch}(c + d\sqrt{x}))^2 dx$$

Optimal(type 4, 310 leaves, 21 steps):

$$\begin{aligned}
& - \frac{2 b^2 x^2}{d} + \frac{2 a^2 x^5 / 2}{5} - \frac{8 a b x^2 \operatorname{arctanh}(e^{c+d\sqrt{x}})}{d} - \frac{2 b^2 x^2 \operatorname{coth}(c + d\sqrt{x})}{d} + \frac{8 b^2 x^3 / 2 \ln(1 - e^{2c+2d\sqrt{x}})}{d^2} - \frac{16 a b x^3 / 2 \operatorname{polylog}(2, -e^{c+d\sqrt{x}})}{d^2} \\
& + \frac{16 a b x^3 / 2 \operatorname{polylog}(2, e^{c+d\sqrt{x}})}{d^2} + \frac{12 b^2 x \operatorname{polylog}(2, e^{2c+2d\sqrt{x}})}{d^3} + \frac{48 a b x \operatorname{polylog}(3, -e^{c+d\sqrt{x}})}{d^3} - \frac{48 a b x \operatorname{polylog}(3, e^{c+d\sqrt{x}})}{d^3} \\
& + \frac{6 b^2 \operatorname{polylog}(4, e^{2c+2d\sqrt{x}})}{d^5} + \frac{96 a b \operatorname{polylog}(5, -e^{c+d\sqrt{x}})}{d^5} - \frac{96 a b \operatorname{polylog}(5, e^{c+d\sqrt{x}})}{d^5} - \frac{12 b^2 \operatorname{polylog}(3, e^{2c+2d\sqrt{x}}) \sqrt{x}}{d^4} \\
& - \frac{96 a b \operatorname{polylog}(4, -e^{c+d\sqrt{x}}) \sqrt{x}}{d^4} + \frac{96 a b \operatorname{polylog}(4, e^{c+d\sqrt{x}}) \sqrt{x}}{d^4}
\end{aligned}$$

Result(type 8, 20 leaves):

$$\int x^3 / 2 (a + b \operatorname{csch}(c + d\sqrt{x}))^2 dx$$

Problem 22: Unable to integrate problem.

$$\int \frac{x^3 / 2}{(a + b \operatorname{csch}(c + d\sqrt{x}))^2} dx$$

Optimal(type 4, 1427 leaves, 43 steps):

$$\begin{aligned}
& \frac{96 b \operatorname{polylog}\left(5, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d^5 \sqrt{a^2+b^2}} - \frac{96 b \operatorname{polylog}\left(5, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d^5 \sqrt{a^2+b^2}} + \frac{48 b^2 \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^5} + \frac{48 b^2 \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^5} \\
& - \frac{48 b^3 \operatorname{polylog}\left(5, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^5} + \frac{48 b^3 \operatorname{polylog}\left(5, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^5} - \frac{2 b^2 x^2}{a^2 (a^2+b^2) d} + \frac{2 x^5 / 2}{5 a^2} - \frac{4 b x^2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d \sqrt{a^2+b^2}} \\
& + \frac{4 b x^2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d \sqrt{a^2+b^2}} - \frac{16 b x^3 / 2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}} + \frac{16 b x^3 / 2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}} \\
& + \frac{48 b x \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 d^3 \sqrt{a^2+b^2}} - \frac{48 b x \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 d^3 \sqrt{a^2+b^2}} - \frac{48 b^2 \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 (a^2+b^2) d^4} \\
& - \frac{48 b^2 \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 (a^2+b^2) d^4} + \frac{48 b^3 \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 (a^2+b^2)^{3/2} d^4} - \frac{48 b^3 \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 (a^2+b^2)^{3/2} d^4} \\
& - \frac{96 b \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 d^4 \sqrt{a^2+b^2}} + \frac{96 b \operatorname{polylog}\left(4, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right) \sqrt{x}}{a^2 d^4 \sqrt{a^2+b^2}} + \frac{8 b^2 x^3 / 2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^2} \\
& + \frac{2 b^3 x^2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d} + \frac{8 b^2 x^3 / 2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^2} - \frac{2 b^3 x^2 \ln\left(1 + \frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d} + \frac{24 b^2 x \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^3} \\
& + \frac{8 b^3 x^3 / 2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^2} + \frac{24 b^2 x \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2) d^3} - \frac{8 b^3 x^3 / 2 \operatorname{polylog}\left(2, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^2} \\
& - \frac{24 b^3 x \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b-\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^3} + \frac{24 b^3 x \operatorname{polylog}\left(3, -\frac{a e^{c+d\sqrt{x}}}{b+\sqrt{a^2+b^2}}\right)}{a^2 (a^2+b^2)^{3/2} d^3} - \frac{2 b^2 x^2 \cosh(c+d\sqrt{x})}{a (a^2+b^2) d (b+a \sinh(c+d\sqrt{x}))}
\end{aligned}$$

Result(type 8, 20 leaves):

$$\int \frac{x^3 / 2}{(a + b \operatorname{csch}(c + d\sqrt{x}))^2} dx$$

Problem 24: Unable to integrate problem.

$$\int \frac{(ex)^{-1+3n}}{a + b \operatorname{csch}(c + dx^n)} dx$$

Optimal(type 4, 404 leaves, 14 steps):

$$\begin{aligned} & \frac{(ex)^{3n}}{3aen} - \frac{b(ex)^{3n} \ln\left(1 + \frac{ae^{c+dx^n}}{b - \sqrt{a^2 + b^2}}\right)}{adenx^n \sqrt{a^2 + b^2}} + \frac{b(ex)^{3n} \ln\left(1 + \frac{ae^{c+dx^n}}{b + \sqrt{a^2 + b^2}}\right)}{adenx^n \sqrt{a^2 + b^2}} - \frac{2b(ex)^{3n} \operatorname{polylog}\left(2, -\frac{ae^{c+dx^n}}{b - \sqrt{a^2 + b^2}}\right)}{ad^2enx^{2n} \sqrt{a^2 + b^2}} \\ & + \frac{2b(ex)^{3n} \operatorname{polylog}\left(2, -\frac{ae^{c+dx^n}}{b + \sqrt{a^2 + b^2}}\right)}{ad^2enx^{2n} \sqrt{a^2 + b^2}} + \frac{2b(ex)^{3n} \operatorname{polylog}\left(3, -\frac{ae^{c+dx^n}}{b - \sqrt{a^2 + b^2}}\right)}{ad^3enx^{3n} \sqrt{a^2 + b^2}} - \frac{2b(ex)^{3n} \operatorname{polylog}\left(3, -\frac{ae^{c+dx^n}}{b + \sqrt{a^2 + b^2}}\right)}{ad^3enx^{3n} \sqrt{a^2 + b^2}} \end{aligned}$$

Result(type 8, 161 leaves):

$$\begin{aligned} & \frac{x e^{(-1+3n) \left(\ln(e) + \ln(x) - \frac{I\pi \operatorname{csgn}(Iex) (-\operatorname{csgn}(Iex) + \operatorname{csgn}(Ie)) (-\operatorname{csgn}(Iex) + \operatorname{csgn}(Ix))}{2} \right)}}{3an} + \int \\ & - \frac{2be^{(-1+3n) \left(\ln(e) + \ln(x) - \frac{I\pi \operatorname{csgn}(Iex) (-\operatorname{csgn}(Iex) + \operatorname{csgn}(Ie)) (-\operatorname{csgn}(Iex) + \operatorname{csgn}(Ix))}{2} \right)} e^{c+de^n \ln(x)}}{a \left(a \left(e^{c+de^n \ln(x)} \right)^2 + 2be^{c+de^n \ln(x)} - a \right)} dx \end{aligned}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \frac{(ex)^{-1+n}}{(a + b \operatorname{csch}(c + dx^n))^2} dx$$

Optimal(type 3, 144 leaves, 8 steps):

$$\frac{(ex)^n}{a^2en} + \frac{2b(2a^2 + b^2)(ex)^n \operatorname{arctanh}\left(\frac{a - b \tanh\left(\frac{c}{2} + \frac{dx^n}{2}\right)}{\sqrt{a^2 + b^2}}\right)}{a^2(a^2 + b^2)^{3/2}denx^n} - \frac{b^2(ex)^n \operatorname{coth}(c + dx^n)}{a(a^2 + b^2)denx^n(a + b \operatorname{csch}(c + dx^n))}$$

Result(type 3, 489 leaves):

$$\frac{x e^{(-1+n) \left(I \operatorname{csgn}(Iex)^3 \pi - I \operatorname{csgn}(Iex)^2 \operatorname{csgn}(Ie) \pi - I \operatorname{csgn}(Iex)^2 \operatorname{csgn}(Ix) \pi + I \operatorname{csgn}(Iex) \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \pi - 2 \ln(x) - 2 \ln(e) \right)}}{a^2n}$$

$$\begin{aligned}
& - \frac{(-1+n) (\text{Icsgn}(Iex)^3 \pi - \text{Icsgn}(Iex)^2 \text{csgn}(Ie) \pi - \text{Icsgn}(Iex)^2 \text{csgn}(Ix) \pi + \text{Icsgn}(Iex) \text{csgn}(Ie) \text{csgn}(Ix) \pi - 2 \ln(x) - 2 \ln(e))}{2 b^2 e} x (-b e^{c+dx^n} + a) \\
& - \frac{1}{a^2 (a^2 + b^2) n e d \sqrt{-a^2 e^{2c} - e^{2c} b^2}} \left(2 b (2 a^2 \right. \\
& \left. + b^2) \right. \\
& - \frac{1}{e} \frac{\pi n \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Iex)}{2} \frac{1}{e^2} \frac{\pi n \text{csgn}(Ie) \text{csgn}(Iex)^2}{2} \frac{1}{e^2} \frac{\pi n \text{csgn}(Ix) \text{csgn}(Iex)^2}{2} - \frac{1}{e} \frac{\pi n \text{csgn}(Iex)^3}{2} \frac{1}{e^2} \frac{\pi \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Iex)}{2} - \frac{1}{e} \frac{\pi \text{csgn}(Ie) \text{csgn}(Iex)^2}{2} \\
& \left. - \frac{1}{e} \frac{\pi \text{csgn}(Ix) \text{csgn}(Iex)^2}{2} \frac{1}{e^2} \frac{\pi \text{csgn}(Iex)^3}{2} e^n e^c \arctan \left(\frac{2 a e^{2c+dx^n} + 2 e^c b}{2 \sqrt{-a^2 e^{2c} - e^{2c} b^2}} \right) \right)
\end{aligned}$$

Test results for the 48 problems in "6.6.3 Hyperbolic cosecant functions.txt"

Problem 5: Unable to integrate problem.

$$\int (b \operatorname{csch}(dx+c))^3 / 2 \, dx$$

Optimal(type 4, 105 leaves, 3 steps):

$$- \frac{2 b \cosh(dx+c) \sqrt{b \operatorname{csch}(dx+c)}}{d} + \frac{2 I b^2 \sqrt{\sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right)^2} \operatorname{EllipticE}\left(\cos\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right) d \sqrt{b \operatorname{csch}(dx+c)} \sqrt{I \sinh(dx+c)}}$$

Result(type 8, 12 leaves):

$$\int (b \operatorname{csch}(dx+c))^3 / 2 \, dx$$

Problem 6: Unable to integrate problem.

$$\int \sqrt{b \operatorname{csch}(dx+c)} \, dx$$

Optimal(type 4, 79 leaves, 2 steps):

$$\frac{2 I \sqrt{\sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right)^2} \operatorname{EllipticF}\left(\cos\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right), \sqrt{2}\right) \sqrt{b \operatorname{csch}(dx+c)} \sqrt{I \sinh(dx+c)}}{\sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right) d}$$

Result(type 8, 12 leaves):

$$\int \sqrt{b \operatorname{csch}(dx+c)} \, dx$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{b \operatorname{csch}(dx+c)}} \, dx$$

Optimal(type 4, 79 leaves, 2 steps):

$$\frac{2I \sqrt{\sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{Idx}{2}\right)^2} \operatorname{EllipticE}\left(\cos\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{Idx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{Idx}{2}\right) d \sqrt{b \operatorname{csch}(dx+c)} \sqrt{I \sinh(dx+c)}}$$

Result(type 4, 226 leaves):

$$\frac{\frac{\sqrt{2}}{d \sqrt{\frac{b e^{dx+c}}{(e^{dx+c})^2 - 1}}} - \frac{1}{d \sqrt{\frac{b e^{dx+c}}{(e^{dx+c})^2 - 1}} \left((e^{dx+c})^2 - 1 \right)}{\left(\frac{2 \left((e^{dx+c})^2 b - b \right)}{b \sqrt{e^{dx+c} \left((e^{dx+c})^2 b - b \right)}} \right)} \left(\frac{\sqrt{e^{dx+c} + 1} \sqrt{-2 e^{dx+c} + 2} \sqrt{-e^{dx+c}} \left(-2 \operatorname{EllipticE}\left(\sqrt{e^{dx+c} + 1}, \frac{\sqrt{2}}{2}\right) + \operatorname{EllipticF}\left(\sqrt{e^{dx+c} + 1}, \frac{\sqrt{2}}{2}\right) \right)}{\sqrt{b (e^{dx+c})^3 - b e^{dx+c}}} \right) \sqrt{2} \sqrt{b e^{dx+c} \left((e^{dx+c})^2 - 1 \right)}}{1}$$

Problem 8: Unable to integrate problem.

$$\int \frac{1}{(b \operatorname{csch}(dx+c))^3 / 2} \, dx$$

Optimal(type 4, 107 leaves, 3 steps):

$$\frac{2 \cosh(dx+c)}{3 b d \sqrt{b \operatorname{csch}(dx+c)}} - \frac{2I \sqrt{\sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{Idx}{2}\right)^2} \operatorname{EllipticF}\left(\cos\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{Idx}{2}\right), \sqrt{2}\right) \sqrt{b \operatorname{csch}(dx+c)} \sqrt{I \sinh(dx+c)}}{3 \sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{Idx}{2}\right) b^2 d}$$

Result(type 8, 12 leaves):

$$\int \frac{1}{(b \operatorname{csch}(dx+c))^3 / 2} \, dx$$

Problem 9: Unable to integrate problem.

$$\int \frac{1}{(b \operatorname{csch}(dx+c))^5 / 2} dx$$

Optimal(type 4, 107 leaves, 3 steps):

$$\frac{2 \cosh(dx+c)}{5 b d (b \operatorname{csch}(dx+c))^3 / 2} - \frac{6 I \sqrt{\sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right)^2} \operatorname{EllipticE}\left(\cos\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{Ic}{2} + \frac{\pi}{4} + \frac{I dx}{2}\right) b^2 d \sqrt{b \operatorname{csch}(dx+c)} \sqrt{I \sinh(dx+c)}}$$

Result(type 8, 12 leaves):

$$\int \frac{1}{(b \operatorname{csch}(dx+c))^5 / 2} dx$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int (-\operatorname{csch}(x)^2)^3 / 2 dx$$

Optimal(type 3, 18 leaves, 3 steps):

$$\frac{\arcsin(\operatorname{coth}(x))}{2} + \frac{\operatorname{coth}(x) \sqrt{-\operatorname{csch}(x)^2}}{2}$$

Result(type 3, 98 leaves):

$$\frac{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}} (e^{2x}+1)}{e^{2x}-1} - \frac{e^{-x} (e^{2x}-1) \sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}} \ln(1+e^x)}{2} + \frac{e^{-x} (e^{2x}-1) \sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}} \ln(e^x-1)}{2}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int (a \operatorname{csch}(x)^2)^5 / 2 dx$$

Optimal(type 3, 49 leaves, 5 steps):

$$-\frac{3 a^5 / 2 \operatorname{arctanh}\left(\frac{\operatorname{coth}(x) \sqrt{a}}{\sqrt{a \operatorname{csch}(x)^2}}\right)}{8} - \frac{a \operatorname{coth}(x) (a \operatorname{csch}(x)^2)^3 / 2}{4} + \frac{3 a^2 \operatorname{coth}(x) \sqrt{a \operatorname{csch}(x)^2}}{8}$$

Result(type 3, 122 leaves):

$$\frac{a^2 \sqrt{\frac{a e^{2x}}{(e^{2x}-1)^2}} (3 e^{6x} - 11 e^{4x} - 11 e^{2x} + 3)}{4 (e^{2x}-1)^3} - \frac{3 a^2 e^{-x} (e^{2x}-1) \sqrt{\frac{a e^{2x}}{(e^{2x}-1)^2}} \ln(1+e^x)}{8} + \frac{3 a^2 e^{-x} (e^{2x}-1) \sqrt{\frac{a e^{2x}}{(e^{2x}-1)^2}} \ln(e^x-1)}{8}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a \operatorname{csch}(x)^2)^{7/2}} dx$$

Optimal(type 3, 58 leaves, 5 steps):

$$\frac{\operatorname{coth}(x)}{7 (a \operatorname{csch}(x)^2)^{7/2}} - \frac{6 \operatorname{coth}(x)}{35 a (a \operatorname{csch}(x)^2)^{5/2}} + \frac{8 \operatorname{coth}(x)}{35 a^2 (a \operatorname{csch}(x)^2)^{3/2}} - \frac{16 \operatorname{coth}(x)}{35 a^3 \sqrt{a \operatorname{csch}(x)^2}}$$

Result(type 3, 261 leaves):

$$\begin{aligned} & \frac{e^{8x}}{896 a^3 (e^{2x} - 1) \sqrt{\frac{a e^{2x}}{(e^{2x} - 1)^2}}} - \frac{7 e^{6x}}{640 a^3 (e^{2x} - 1) \sqrt{\frac{a e^{2x}}{(e^{2x} - 1)^2}}} + \frac{7 e^{4x}}{128 a^3 (e^{2x} - 1) \sqrt{\frac{a e^{2x}}{(e^{2x} - 1)^2}}} - \frac{35 e^{2x}}{128 a^3 (e^{2x} - 1) \sqrt{\frac{a e^{2x}}{(e^{2x} - 1)^2}}} \\ & - \frac{35}{128 a^3 (e^{2x} - 1) \sqrt{\frac{a e^{2x}}{(e^{2x} - 1)^2}}} + \frac{7 e^{-2x}}{128 a^3 (e^{2x} - 1) \sqrt{\frac{a e^{2x}}{(e^{2x} - 1)^2}}} - \frac{7 e^{-4x}}{640 a^3 (e^{2x} - 1) \sqrt{\frac{a e^{2x}}{(e^{2x} - 1)^2}}} + \frac{e^{-6x}}{896 a^3 (e^{2x} - 1) \sqrt{\frac{a e^{2x}}{(e^{2x} - 1)^2}}} \end{aligned}$$

Problem 13: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a \operatorname{csch}(x)^3}} dx$$

Optimal(type 4, 72 leaves, 4 steps):

$$\frac{2 \operatorname{coth}(x)}{3 \sqrt{a \operatorname{csch}(x)^3}} - \frac{2 \operatorname{Icsch}(x)^2 \sqrt{\sin\left(\frac{\pi}{4} + \frac{Ix}{2}\right)^2} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{Ix}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{Isinh}(x)}}{3 \sin\left(\frac{\pi}{4} + \frac{Ix}{2}\right) \sqrt{a \operatorname{csch}(x)^3}}$$

Result(type 8, 10 leaves):

$$\int \frac{1}{\sqrt{a \operatorname{csch}(x)^3}} dx$$

Problem 14: Unable to integrate problem.

$$\int \frac{1}{(a \operatorname{csch}(x)^3)^{5/2}} dx$$

Optimal(type 4, 133 leaves, 7 steps):

$$-\frac{26 \operatorname{coth}(x)}{77 a^2 \sqrt{a \operatorname{csch}(x)^3}} + \frac{78 \cosh(x) \sinh(x)}{385 a^2 \sqrt{a \operatorname{csch}(x)^3}} - \frac{26 \cosh(x) \sinh(x)^3}{165 a^2 \sqrt{a \operatorname{csch}(x)^3}} + \frac{2 \cosh(x) \sinh(x)^5}{15 a^2 \sqrt{a \operatorname{csch}(x)^3}}$$

$$+ \frac{26 \operatorname{Icsch}(x)^2 \sqrt{\sin\left(\frac{\pi}{4} + \frac{\operatorname{I}x}{2}\right)^2} \operatorname{EllipticF}\left(\cos\left(\frac{\pi}{4} + \frac{\operatorname{I}x}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{I} \sinh(x)}}{77 \sin\left(\frac{\pi}{4} + \frac{\operatorname{I}x}{2}\right) a^2 \sqrt{a \operatorname{csch}(x)^3}}$$

Result(type 8, 10 leaves):

$$\int \frac{1}{(a \operatorname{csch}(x)^3)^{5/2}} dx$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a \operatorname{csch}(x)^4)^{3/2}} dx$$

Optimal(type 3, 70 leaves, 5 steps):

$$\frac{5 \operatorname{coth}(x)}{16 a \sqrt{a \operatorname{csch}(x)^4}} - \frac{5 x \operatorname{csch}(x)^2}{16 a \sqrt{a \operatorname{csch}(x)^4}} - \frac{5 \operatorname{cosh}(x) \sinh(x)}{24 a \sqrt{a \operatorname{csch}(x)^4}} + \frac{\operatorname{cosh}(x) \sinh(x)^3}{6 a \sqrt{a \operatorname{csch}(x)^4}}$$

Result(type 3, 229 leaves):

$$\begin{aligned} & - \frac{5 e^{2x} x}{16 a (e^{2x} - 1)^2 \sqrt{\frac{a e^{4x}}{(e^{2x} - 1)^4}}} + \frac{e^{8x}}{384 a (e^{2x} - 1)^2 \sqrt{\frac{a e^{4x}}{(e^{2x} - 1)^4}}} - \frac{3 e^{6x}}{128 a (e^{2x} - 1)^2 \sqrt{\frac{a e^{4x}}{(e^{2x} - 1)^4}}} + \frac{15 e^{4x}}{128 a (e^{2x} - 1)^2 \sqrt{\frac{a e^{4x}}{(e^{2x} - 1)^4}}} \\ & - \frac{15}{128 a (e^{2x} - 1)^2 \sqrt{\frac{a e^{4x}}{(e^{2x} - 1)^4}}} + \frac{3 e^{-2x}}{128 a (e^{2x} - 1)^2 \sqrt{\frac{a e^{4x}}{(e^{2x} - 1)^4}}} - \frac{e^{-4x}}{384 a (e^{2x} - 1)^2 \sqrt{\frac{a e^{4x}}{(e^{2x} - 1)^4}}} \end{aligned}$$

Problem 18: Unable to integrate problem.

$$\int \sqrt{a - \operatorname{I} a \operatorname{csch}(dx + c)} dx$$

Optimal(type 3, 33 leaves, 2 steps):

$$\frac{2 \operatorname{arctanh}\left(\frac{\operatorname{coth}(dx + c) \sqrt{a}}{\sqrt{a - \operatorname{I} a \operatorname{csch}(dx + c)}}\right) \sqrt{a}}{d}$$

Result(type 8, 16 leaves):

$$\int \sqrt{a - \operatorname{I} a \operatorname{csch}(dx + c)} dx$$

Problem 19: Unable to integrate problem.

$$\int \sqrt{3 - 3 \operatorname{Icsch}(x)} \, dx$$

Optimal(type 3, 18 leaves, 2 steps):

$$2 \operatorname{arctanh}\left(\frac{\operatorname{coth}(x)}{\sqrt{1 - \operatorname{Icsch}(x)}}\right) \sqrt{3}$$

Result(type 8, 11 leaves):

$$\int \sqrt{3 - 3 \operatorname{Icsch}(x)} \, dx$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^2}{1 + \operatorname{csch}(x)} \, dx$$

Optimal(type 3, 29 leaves, 5 steps):

$$\frac{3 \operatorname{Ix}}{2} + 2 \operatorname{cosh}(x) - \frac{3 \operatorname{Icosh}(x) \sinh(x)}{2} - \frac{\operatorname{cosh}(x) \sinh(x)}{1 + \operatorname{csch}(x)}$$

Result(type 3, 95 leaves):

$$\begin{aligned} & -\frac{3 \operatorname{I} \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{2} - \frac{\operatorname{I}}{2 \left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} - \frac{1}{\tanh\left(\frac{x}{2}\right) - 1} - \frac{\operatorname{I}}{2 \left(\tanh\left(\frac{x}{2}\right) - 1\right)} - \frac{2 \operatorname{I}}{\tanh\left(\frac{x}{2}\right) - 1} + \frac{\operatorname{I}}{2 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} \\ & + \frac{3 \operatorname{I} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{2} + \frac{1}{\tanh\left(\frac{x}{2}\right) + 1} - \frac{\operatorname{I}}{2 \left(\tanh\left(\frac{x}{2}\right) + 1\right)} \end{aligned}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)}{1 + \operatorname{csch}(x)} \, dx$$

Optimal(type 3, 18 leaves, 4 steps):

$$x - 2 \operatorname{Icosh}(x) - \frac{\operatorname{cosh}(x)}{1 + \operatorname{csch}(x)}$$

Result(type 3, 50 leaves):

$$\frac{\operatorname{I}}{\tanh\left(\frac{x}{2}\right) - 1} - \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - \frac{2}{\tanh\left(\frac{x}{2}\right) - 1} - \frac{\operatorname{I}}{\tanh\left(\frac{x}{2}\right) + 1} + \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^4}{1 + \operatorname{csch}(x)} dx$$

Optimal (type 3, 21 leaves, 7 steps):

$$-\frac{\operatorname{sech}(x)^5}{5} - \frac{1 \tanh(x)^3}{3} + \frac{1 \tanh(x)^5}{5}$$

Result (type 3, 92 leaves):

$$\begin{aligned} & -\frac{4I}{3 \left(\tanh\left(\frac{x}{2}\right) - I \right)^3} + \frac{3I}{8 \left(\tanh\left(\frac{x}{2}\right) - I \right)} + \frac{2I}{5 \left(\tanh\left(\frac{x}{2}\right) - I \right)^5} + \frac{1}{\left(\tanh\left(\frac{x}{2}\right) - I \right)^4} - \frac{1}{\left(\tanh\left(\frac{x}{2}\right) - I \right)^2} + \frac{I}{6 \left(\tanh\left(\frac{x}{2}\right) + I \right)^3} \\ & - \frac{3I}{8 \left(\tanh\left(\frac{x}{2}\right) + I \right)} - \frac{1}{4 \left(\tanh\left(\frac{x}{2}\right) + I \right)^2} \end{aligned}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(x)^5}{a + b \operatorname{csch}(x)} dx$$

Optimal (type 3, 94 leaves, 5 steps):

$$-\frac{b(a^2 + b^2)^2 \ln(b + a \sinh(x))}{a^6} + \frac{(a^2 + b^2)^2 \sinh(x)}{a^5} - \frac{b(2a^2 + b^2) \sinh(x)^2}{2a^4} + \frac{(2a^2 + b^2) \sinh(x)^3}{3a^3} - \frac{b \sinh(x)^4}{4a^2} + \frac{\sinh(x)^5}{5a}$$

Result (type 3, 599 leaves):

$$\begin{aligned} & \frac{b \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a^2} + \frac{2b^3 \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a^4} + \frac{b^5 \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a^6} - \frac{b \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{a^2} \\ & - \frac{2b^3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{a^4} - \frac{b^5 \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{a^6} - \frac{b}{4a^2 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^4} + \frac{b}{2a^2 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} \\ & - \frac{b^2}{3a^3 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} - \frac{9b}{8a^2 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} + \frac{b^2}{2a^3 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} - \frac{b^3}{2a^4 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} + \frac{7b}{8a^2 \left(\tanh\left(\frac{x}{2}\right) + 1\right)} \\ & - \frac{2b^2}{a^3 \left(\tanh\left(\frac{x}{2}\right) + 1\right)} + \frac{b^3}{2a^4 \left(\tanh\left(\frac{x}{2}\right) + 1\right)} - \frac{b^4}{a^5 \left(\tanh\left(\frac{x}{2}\right) + 1\right)} + \frac{b \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a^2} + \frac{2b^3 \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a^4} \\ & + \frac{b^5 \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a^6} - \frac{7b}{8a^2 \left(\tanh\left(\frac{x}{2}\right) - 1\right)} - \frac{2b^2}{a^3 \left(\tanh\left(\frac{x}{2}\right) - 1\right)} - \frac{b^3}{2a^4 \left(\tanh\left(\frac{x}{2}\right) - 1\right)} - \frac{b^4}{a^5 \left(\tanh\left(\frac{x}{2}\right) - 1\right)} \end{aligned}$$

$$\begin{aligned}
& - \frac{9b}{8a^2 \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^2} - \frac{b^2}{2a^3 \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^2} - \frac{b^3}{2a^4 \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^2} - \frac{b}{2a^2 \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^3} - \frac{b^2}{3a^3 \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^3} \\
& - \frac{b}{4a^2 \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^4} + \frac{1}{2a \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^4} + \frac{7}{8a \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^2} - \frac{1}{a \left(\tanh\left(\frac{x}{2}\right) + 1 \right)} - \frac{11}{12a \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^3} \\
& - \frac{11}{12a \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^3} - \frac{1}{5a \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^5} - \frac{1}{5a \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^5} - \frac{1}{2a \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^4} - \frac{7}{8a \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^2} \\
& - \frac{1}{a \left(\tanh\left(\frac{x}{2}\right) - 1 \right)}
\end{aligned}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^5}{a + b \operatorname{csch}(x)} dx$$

Optimal (type 3, 135 leaves, 7 steps):

$$\begin{aligned}
& - \frac{a(3Ia + b) \ln(I - \sinh(x))}{16(a - Ib)^3} + \frac{a(3a + Ib) \ln(I + \sinh(x))}{16(Ia - b)^3} - \frac{a^4 b \ln(b + a \sinh(x))}{(a^2 + b^2)^3} - \frac{\operatorname{sech}(x)^4 (b - a \sinh(x))}{4(a^2 + b^2)} \\
& - \frac{\operatorname{sech}(x)^2 (4a^2 b - a(3a^2 - b^2) \sinh(x))}{8(a^2 + b^2)^2}
\end{aligned}$$

Result (type 3, 1167 leaves):

$$\begin{aligned}
& - \frac{a^4 b \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2)} - \frac{5 \tanh\left(\frac{x}{2}\right)^7 a^5}{4(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{2 \tanh\left(\frac{x}{2}\right)^6 b^5}{(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& + \frac{3 \tanh\left(\frac{x}{2}\right)^5 a^5}{4(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{3 \tanh\left(\frac{x}{2}\right)^3 a^5}{4(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& + \frac{2 \tanh\left(\frac{x}{2}\right)^2 b^5}{(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{5 \tanh\left(\frac{x}{2}\right) a^5}{4(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{a^4 b \ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)}{(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2)} \\
& - \frac{3 \arctan\left(\tanh\left(\frac{x}{2}\right)\right) a^3 b^2}{2(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2)} - \frac{\arctan\left(\tanh\left(\frac{x}{2}\right)\right) a b^4}{4(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2)} + \frac{3 \arctan\left(\tanh\left(\frac{x}{2}\right)\right) a^5}{4(a^4 + 2a^2 b^2 + b^4)(a^2 + b^2)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{4 \tanh\left(\frac{x}{2}\right)^2 a^4 b}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{6 \tanh\left(\frac{x}{2}\right)^2 a^2 b^3}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& + \frac{3 \tanh\left(\frac{x}{2}\right) a^3 b^2}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{\tanh\left(\frac{x}{2}\right) a b^4}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& - \frac{3 \tanh\left(\frac{x}{2}\right)^7 a^3 b^2}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{\tanh\left(\frac{x}{2}\right)^7 a b^4}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& + \frac{4 \tanh\left(\frac{x}{2}\right)^6 a^4 b}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{6 \tanh\left(\frac{x}{2}\right)^6 a^2 b^3}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& + \frac{5 \tanh\left(\frac{x}{2}\right)^5 a^3 b^2}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{7 \tanh\left(\frac{x}{2}\right)^5 a b^4}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& + \frac{4 \tanh\left(\frac{x}{2}\right)^4 a^4 b}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{4 \tanh\left(\frac{x}{2}\right)^4 a^2 b^3}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& - \frac{5 \tanh\left(\frac{x}{2}\right)^3 a^3 b^2}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{7 \tanh\left(\frac{x}{2}\right)^3 a b^4}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4}
\end{aligned}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)^5}{a + b \operatorname{csch}(x)} dx$$

Optimal(type 3, 184 leaves, 11 steps):

$$-\frac{b^5 \arctan(\sinh(x))}{(a^2 + b^2)^3} - \frac{b^3 \arctan(\sinh(x))}{2 (a^2 + b^2)^2} - \frac{3 b \arctan(\sinh(x))}{8 (a^2 + b^2)} + \frac{b^6 \ln(a + b \operatorname{csch}(x))}{a (a^2 + b^2)^3} + \frac{\ln(\sinh(x))}{a} - \frac{a (a^4 + 3 a^2 b^2 + 3 b^4) \ln(\tanh(x))}{(a^2 + b^2)^3}$$

$$+ \frac{3 b \operatorname{sech}(x) \tanh(x)}{8 (a^2 + b^2)} - \frac{(a (a^2 + 2 b^2) - b^3 \operatorname{csch}(x)) \tanh(x)^2}{2 (a^2 + b^2)^2} - \frac{(a - b \operatorname{csch}(x)) \tanh(x)^4}{4 (a^2 + b^2)}$$

Result(type 3, 1322 leaves):

$$\begin{aligned} & \frac{b^6 \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2 a \tanh\left(\frac{x}{2}\right) - b\right)}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) a} - \frac{3 \arctan\left(\tanh\left(\frac{x}{2}\right)\right) a^4 b}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2)} - \frac{5 \arctan\left(\tanh\left(\frac{x}{2}\right)\right) a^2 b^3}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2)} \\ & - \frac{7 \tanh\left(\frac{x}{2}\right)^7 b^5}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{2 \tanh\left(\frac{x}{2}\right)^6 a^5}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\ & - \frac{15 \tanh\left(\frac{x}{2}\right)^5 b^5}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{8 \tanh\left(\frac{x}{2}\right)^4 a^5}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\ & - \frac{15 \tanh\left(\frac{x}{2}\right)^3 b^5}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{2 \tanh\left(\frac{x}{2}\right)^2 a^5}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\ & + \frac{7 \tanh\left(\frac{x}{2}\right) b^5}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right) a^3 b^2}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2)} + \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right) a b^4}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2)} \\ & - \frac{3 \tanh\left(\frac{x}{2}\right)^7 a^4 b}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{5 \tanh\left(\frac{x}{2}\right)^7 a^2 b^3}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\ & - \frac{6 \tanh\left(\frac{x}{2}\right)^6 a^3 b^2}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{4 \tanh\left(\frac{x}{2}\right)^6 a b^4}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\ & - \frac{13 \tanh\left(\frac{x}{2}\right)^5 a^2 b^3}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{11 \tanh\left(\frac{x}{2}\right)^5 a^4 b}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \end{aligned}$$

$$\begin{aligned}
& - \frac{20 \tanh\left(\frac{x}{2}\right)^4 a^3 b^2}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{12 \tanh\left(\frac{x}{2}\right)^4 a b^4}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& + \frac{13 \tanh\left(\frac{x}{2}\right)^3 a^2 b^3}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{11 \tanh\left(\frac{x}{2}\right)^3 a^4 b}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& - \frac{6 \tanh\left(\frac{x}{2}\right)^2 a^3 b^2}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} - \frac{4 \tanh\left(\frac{x}{2}\right)^2 a b^4}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} \\
& + \frac{3 \tanh\left(\frac{x}{2}\right) a^4 b}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{5 \tanh\left(\frac{x}{2}\right) a^2 b^3}{2 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2) \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^4} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right) a^5}{(a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2)} \\
& - \frac{15 \arctan\left(\tanh\left(\frac{x}{2}\right)\right) b^5}{4 (a^4 + 2 a^2 b^2 + b^4) (a^2 + b^2)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a}
\end{aligned}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(x)^2}{a + b \operatorname{csch}(x)} dx$$

Optimal (type 3, 51 leaves, 8 steps):

$$\frac{x}{a} - \frac{\operatorname{arctanh}(\cosh(x))}{b} + \frac{2 \operatorname{arctanh}\left(\frac{a - b \tanh\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2}}\right) \sqrt{a^2 + b^2}}{a b}$$

Result (type 3, 109 leaves):

$$\begin{aligned}
& - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a} - \frac{2 a \operatorname{arctanh}\left(\frac{2 b \tanh\left(\frac{x}{2}\right) - 2 a}{2 \sqrt{a^2 + b^2}}\right)}{b \sqrt{a^2 + b^2}} - \frac{2 b \operatorname{arctanh}\left(\frac{2 b \tanh\left(\frac{x}{2}\right) - 2 a}{2 \sqrt{a^2 + b^2}}\right)}{a \sqrt{a^2 + b^2}} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a}
\end{aligned}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(x)^3}{a + b \operatorname{csch}(x)} dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$-\frac{\operatorname{csch}(x)}{b} + \left(\frac{1}{a} + \frac{a}{b^2} \right) \ln(a + b \operatorname{csch}(x)) + \frac{\ln(\sinh(x))}{a}$$

Result (type 3, 105 leaves):

$$\frac{\tanh\left(\frac{x}{2}\right)}{2b} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a} + \frac{a \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{b^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{a} - \frac{1}{2b \tanh\left(\frac{x}{2}\right)}$$

$$- \frac{a \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a}$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(x)^5}{a + b \operatorname{csch}(x)} dx$$

Optimal (type 3, 66 leaves, 3 steps):

$$-\frac{(a^2 + 2b^2) \operatorname{csch}(x)}{b^3} + \frac{a \operatorname{csch}(x)^2}{2b^2} - \frac{\operatorname{csch}(x)^3}{3b} + \frac{(a^2 + b^2)^2 \ln(a + b \operatorname{csch}(x))}{ab^4} + \frac{\ln(\sinh(x))}{a}$$

Result (type 3, 218 leaves):

$$\frac{\tanh\left(\frac{x}{2}\right)^3}{24b} + \frac{\tanh\left(\frac{x}{2}\right)^2 a}{8b^2} + \frac{a^2 \tanh\left(\frac{x}{2}\right)}{2b^3} + \frac{7 \tanh\left(\frac{x}{2}\right)}{8b} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a} + \frac{a^3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{b^4}$$

$$+ \frac{2a \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{b^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{a} - \frac{1}{24b \tanh\left(\frac{x}{2}\right)^3} - \frac{a^2}{2b^3 \tanh\left(\frac{x}{2}\right)} - \frac{7}{8b \tanh\left(\frac{x}{2}\right)}$$

$$+ \frac{a}{8b^2 \tanh\left(\frac{x}{2}\right)^2} - \frac{a^3 \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^4} - \frac{2a \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(x)^7}{a + b \operatorname{csch}(x)} dx$$

Optimal (type 3, 111 leaves, 3 steps):

$$-\frac{(a^4 + 3a^2b^2 + 3b^4) \operatorname{csch}(x)}{b^5} + \frac{a(a^2 + 3b^2) \operatorname{csch}(x)^2}{2b^4} - \frac{(a^2 + 3b^2) \operatorname{csch}(x)^3}{3b^3} + \frac{a \operatorname{csch}(x)^4}{4b^2} - \frac{\operatorname{csch}(x)^5}{5b} + \frac{(a^2 + b^2)^3 \ln(a + b \operatorname{csch}(x))}{ab^6} + \frac{\ln(\sinh(x))}{a}$$

Result (type 3, 387 leaves):

$$\begin{aligned} & \frac{a^5 \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{b^6} - \frac{a^2}{24b^3 \tanh\left(\frac{x}{2}\right)^3} - \frac{a^4}{2b^5 \tanh\left(\frac{x}{2}\right)} + \frac{a^3}{8b^4 \tanh\left(\frac{x}{2}\right)^2} - \frac{a^5 \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^6} + \frac{a}{64b^2 \tanh\left(\frac{x}{2}\right)^4} \\ & + \frac{\tanh\left(\frac{x}{2}\right)^4 a}{64b^2} + \frac{a^2 \tanh\left(\frac{x}{2}\right)^3}{24b^3} + \frac{\tanh\left(\frac{x}{2}\right)^2 a^3}{8b^4} + \frac{a^4 \tanh\left(\frac{x}{2}\right)}{2b^5} - \frac{1}{160b \tanh\left(\frac{x}{2}\right)^5} + \frac{\tanh\left(\frac{x}{2}\right)^5}{160b} + \frac{5 \tanh\left(\frac{x}{2}\right)^2 a}{16b^2} + \frac{11a^2 \tanh\left(\frac{x}{2}\right)}{8b^3} \\ & + \frac{3a^3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{b^4} - \frac{11a^2}{8b^3 \tanh\left(\frac{x}{2}\right)} + \frac{5a}{16b^2 \tanh\left(\frac{x}{2}\right)^2} - \frac{3a^3 \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^4} \\ & + \frac{3a \ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{b^2} - \frac{3a \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2} + \frac{19 \tanh\left(\frac{x}{2}\right)}{16b} - \frac{19}{16b \tanh\left(\frac{x}{2}\right)} + \frac{3 \tanh\left(\frac{x}{2}\right)^3}{32b} - \frac{3}{32b \tanh\left(\frac{x}{2}\right)^3} \\ & + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 b - 2a \tanh\left(\frac{x}{2}\right) - b\right)}{a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a} \end{aligned}$$

Problem 45: Unable to integrate problem.

$$\int \frac{\operatorname{csch}(2 \ln(cx))^3 / 2}{x^4} dx$$

Optimal (type 3, 59 leaves, 6 steps):

$$-\frac{\left(c^4 - \frac{1}{x^4}\right) x \operatorname{csch}(2 \ln(cx))^3 / 2}{2} + \frac{c^6 \left(1 - \frac{1}{c^4 x^4}\right)^3 / 2 x^3 \operatorname{arccsc}(c^2 x^2) \operatorname{csch}(2 \ln(cx))^3 / 2}{2}$$

Result (type 8, 15 leaves):

$$\int \frac{\operatorname{csch}(2 \ln(cx))^3 / 2}{x^4} dx$$

Problem 46: Unable to integrate problem.

$$\int \operatorname{csch}(a + b \ln(cx^n)) \, dx$$

Optimal(type 5, 57 leaves, 4 steps):

$$-\frac{2 e^a x (c x^n)^b \operatorname{hypergeom}\left(\left[1, \frac{b + \frac{1}{n}}{2b}\right], \left[\frac{3}{2} + \frac{1}{2bn}\right], e^{2a} (c x^n)^{2b}\right)}{bn + 1}$$

Result(type 8, 13 leaves):

$$\int \operatorname{csch}(a + b \ln(cx^n)) \, dx$$

Test results for the 10 problems in "6.6.7 (d hyper)^m (a+b (c csch)^n)^p.txt"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{csch}(dx + c))^3} \, dx$$

Optimal(type 3, 142 leaves, 6 steps):

$$\frac{x}{a^3} + \frac{b \operatorname{coth}(dx + c)}{4a(-b + a)d(a - b + b \operatorname{coth}(dx + c))^2} + \frac{(7a - 4b)b \operatorname{coth}(dx + c)}{8a^2(-b + a)^2d(a - b + b \operatorname{coth}(dx + c))^2}$$

$$- \frac{(15a^2 - 20ab + 8b^2) \arctan\left(\frac{\sqrt{-b + a} \tanh(dx + c)}{\sqrt{b}}\right) \sqrt{b}}{8a^3(-b + a)^{5/2}d}$$

Result(type ?, 4583 leaves): Display of huge result suppressed!

Problem 4: Unable to integrate problem.

$$\int (a + b \operatorname{csch}(dx + c))^3 / 2 \, dx$$

Optimal(type 3, 108 leaves, 7 steps):

$$\frac{a^3 / 2 \operatorname{arctanh}\left(\frac{\operatorname{coth}(dx + c) \sqrt{a}}{\sqrt{a - b + b \operatorname{coth}(dx + c)^2}}\right)}{d} - \frac{(3a - b) \operatorname{arctanh}\left(\frac{\operatorname{coth}(dx + c) \sqrt{b}}{\sqrt{a - b + b \operatorname{coth}(dx + c)^2}}\right) \sqrt{b}}{2d} - \frac{b \operatorname{coth}(dx + c) \sqrt{a - b + b \operatorname{coth}(dx + c)^2}}{2d}$$

Result(type 8, 16 leaves):

$$\int (a + b \operatorname{csch}(dx + c))^3 / 2 \, dx$$

Problem 5: Unable to integrate problem.

$$\int \frac{1}{(a + b \operatorname{csch}(dx + c))^7 / 2} \, dx$$

Optimal(type 3, 175 leaves, 7 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\coth(dx+c)\sqrt{a}}{\sqrt{a-b+b\coth(dx+c)^2}}\right)}{a^{7/2}d} + \frac{b\coth(dx+c)}{5a(-b+a)d(a-b+b\coth(dx+c)^2)^{5/2}} + \frac{(9a-5b)b\coth(dx+c)}{15a^2(-b+a)^2d(a-b+b\coth(dx+c)^2)^{3/2}} + \frac{b(33a^2-40ab+15b^2)\coth(dx+c)}{15a^3(-b+a)^3d\sqrt{a-b+b\coth(dx+c)^2}}$$

Result(type 8, 16 leaves):

$$\int \frac{1}{(a+b\operatorname{csch}(dx+c)^2)^{7/2}} dx$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int (1+\operatorname{csch}(x)^2)^{3/2} dx$$

Optimal(type 3, 23 leaves, 4 steps):

$$-\frac{(\coth(x)^2)^{3/2}\tanh(x)}{2} + \ln(\sinh(x))\sqrt{\coth(x)^2}\tanh(x)$$

Result(type 3, 119 leaves):

$$-\frac{(e^{2x}-1)\sqrt{\frac{(e^{2x}+1)^2}{(e^{2x}-1)^2}}x}{e^{2x}+1} - \frac{2\sqrt{\frac{(e^{2x}+1)^2}{(e^{2x}-1)^2}}e^{2x}}{(e^{2x}+1)(e^{2x}-1)} + \frac{(e^{2x}-1)\sqrt{\frac{(e^{2x}+1)^2}{(e^{2x}-1)^2}}\ln(e^{2x}-1)}{e^{2x}+1}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1+\operatorname{csch}(x)^2} dx$$

Optimal(type 3, 12 leaves, 3 steps):

$$\ln(\sinh(x))\sqrt{\coth(x)^2}\tanh(x)$$

Result(type 3, 78 leaves):

$$-\frac{(e^{2x}-1)\sqrt{\frac{(e^{2x}+1)^2}{(e^{2x}-1)^2}}x}{e^{2x}+1} + \frac{(e^{2x}-1)\sqrt{\frac{(e^{2x}+1)^2}{(e^{2x}-1)^2}}\ln(e^{2x}-1)}{e^{2x}+1}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 + \operatorname{csch}(x)^2}} dx$$

Optimal(type 3, 12 leaves, 3 steps):

$$\frac{\operatorname{coth}(x) \ln(\cosh(x))}{\sqrt{\operatorname{coth}(x)^2}}$$

Result(type 3, 78 leaves):

$$-\frac{(e^{2x} + 1)x}{\sqrt{\frac{(e^{2x} + 1)^2}{(e^{2x} - 1)^2}} (e^{2x} - 1)} + \frac{(e^{2x} + 1) \ln(e^{2x} + 1)}{\sqrt{\frac{(e^{2x} + 1)^2}{(e^{2x} - 1)^2}} (e^{2x} - 1)}$$

Problem 9: Unable to integrate problem.

$$\int (1 - \operatorname{csch}(x)^2)^{3/2} dx$$

Optimal(type 3, 39 leaves, 6 steps):

$$2 \arcsin\left(\frac{\operatorname{coth}(x) \sqrt{2}}{2}\right) + \operatorname{arctanh}\left(\frac{\operatorname{coth}(x)}{\sqrt{2 - \operatorname{coth}(x)^2}}\right) + \frac{\operatorname{coth}(x) \sqrt{2 - \operatorname{coth}(x)^2}}{2}$$

Result(type 8, 12 leaves):

$$\int (1 - \operatorname{csch}(x)^2)^{3/2} dx$$

Problem 10: Unable to integrate problem.

$$\int \sqrt{-1 + \operatorname{csch}(x)^2} dx$$

Optimal(type 3, 29 leaves, 6 steps):

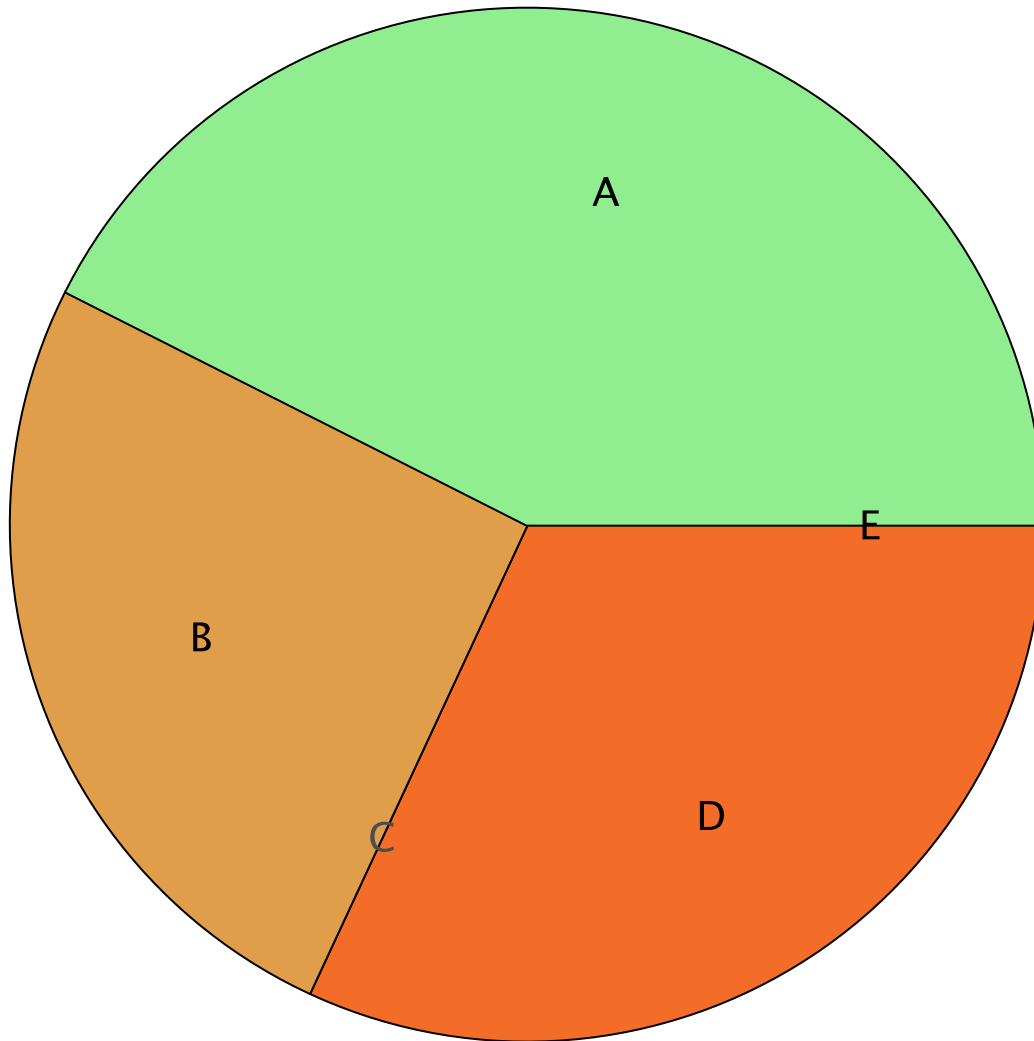
$$-\operatorname{arctan}\left(\frac{\operatorname{coth}(x)}{\sqrt{-2 + \operatorname{coth}(x)^2}}\right) - \operatorname{arctanh}\left(\frac{\operatorname{coth}(x)}{\sqrt{-2 + \operatorname{coth}(x)^2}}\right)$$

Result(type 8, 10 leaves):

$$\int \sqrt{-1 + \operatorname{csch}(x)^2} dx$$

Summary of Integration Test Results

94 integration problems



A - 40 optimal antiderivatives
B - 24 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 30 unable to integrate problems
E - 0 integration timeouts